Grade 9

MATHEMATICS CONTENT BOOKLET: TARGETED SUPPORT

Term 3

A MESSAGE FROM THE NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers,
This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE)! We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

What is NECT?
In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education and to help the DBE reach the NDP goals.

The NECT has successfully brought together groups of relevant people so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

What are the Learning programmes?
One of the programmes that the NECT implements on behalf of the DBE is the ‘District Development Programme’. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). Curriculum learning programmes were developed for Maths, Science and Language teachers in FSS who received training and support on their implementation. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers. The FSS helped the DBE trial the NECT learning programmes so that they could be improved and used by many more teachers. NECT has already begun this embedding process.

Everyone using the learning programmes comes from one of these groups; but you are now brought together in the spirit of collaboration that defines the manner in which the NECT works. Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let’s work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

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INTRODUCTION: THREE PRINCIPLES OF TEACHING MATHEMATICS

PRINCIPLE 1: TEACHING MATHEMATICS DEVELOPMENTALLY

What is developmental teaching and what are the benefits of such an approach?

- The human mind develops through phases or stages which require learning in a certain way and which determine whether children are ready to learn something or not.
- If learners are not ready to learn something, it may be due to the fact that they have not reached that level of development yet or they have missed something previously.
- The idea that children's thinking develop from concrete to abstract, comes from Piaget (1969). We adopted a refined version of that idea though, which works very well for mathematics teaching, namely a "concrete-representational-abstract" classification (Miller & Mercer, 1993).
- It is not possible in all cases or for all topics to follow the "concrete-representational-abstract" sequence exactly, but at the primary level it is possible in many topics and is especially valuable in establishing new concepts.
- This classification gives a tool in the hands of the teacher to develop children's mathematical thinking but also to fall back to a previous phase if it is clear that the learner has missed something previously.
- The need for concrete experiences and the use of concrete objects in learning, may pass as learners develop past the Foundation Phase. However, the representational and abstract development phases are both very much present in learning mathematics at the Intermediate and Senior Phase.

How can this approach be implemented practically?

The table on page 7 illustrates how a multi-modal approach to mathematics teaching may be implemented practically, with examples from several content areas.

What does this look like in the booklet?

Throughout the booklets, within the lesson plans for the Foundation Phase and within the topics at the Intermediate/Senior Phase, suggestions are made to implement this principle in the classroom:

- Where applicable, the initial concrete way of teaching and learning the concept is suggested and educational resources provided at the end of the lesson plan or topic to assist teachers in introducing the idea concretely.
- In most cases pictures (semi-concrete) and/or diagrams (semi-abstract) are provided, either at the clarification of terminology section, within the topic or lesson plan itself or at the end of the lesson plan or topic as an educational resource.
- In all cases the symbolic (abstract) way of teaching and learning the concept, is provided, since this is, developmentally speaking, where we primarily aim to be when learners master mathematics.

PRINCIPLE 2: TEACHING MATHEMATICS MULTI-MODALLY

What is multi-modal teaching and what are the benefits of such an approach?

- We suggest a rhythm of teaching any mathematical topic by way of "saying it, showing it and symbolising it". This approach can be called multi-modal teaching and links in a significant way to the developmental phases above.
- Multi-modal teaching includes speaking about a matter verbally (auditory mode), showing it in a picture or a diagram (visual mode) and writing it in words or numbers (symbolic mode).
- For multi-modal teaching, the same learning material is presented in various modalities: by an explanation using spoken words (auditory), by showing pictures or diagrams (visual) and by writing words and numbers (symbolic).
- Modal preferences amongst learners vary significantly and learning takes place more successfully when they receive, study and present their learning in the mode of their preference, either auditory, visually or symbolically. Although individual learners prefer one mode above another, the exposure to all three of these modes enhance their learning.

How can this approach be implemented practically?

The table on page 7 illustrates how a multi-modal approach to mathematics teaching may be implemented practically, with examples from several content areas.

What does this look like in the booklet?

Throughout the booklets, within the lesson plans for the Foundation Phase and within the topics at the Intermediate/Senior Phase, suggestions are made to implement this principle in the classroom:

- The verbal explanations under each topic and within each lesson plan, provide the "say it" or auditory mode.
- The pictures and diagrams provide suggestions for the "show it" mode (visual mode).
- The calculations, exercises and assessments under each topic and within each lesson plan, provide the "symbol it" or symbolic mode of representation.

PRINCIPLE 3: SEQUENTIAL TEACHING

What is sequential teaching and what are the benefits of such an approach?

- Learners with weak basic skills in mathematics will find future topics increasingly difficult. A solid foundation is required for a good fundamental understanding.
- In order to build a solid foundation in maths the approach to teaching needs to be systematic. Teaching concepts out of sequence can lead to difficulties in grasping concepts.
- Teaching in a systematic way (according to CAPS) allows learners to progressively build understandings, skills and confidence.
- A learner needs to be confident in the principles of a topic before he/she is expected to apply the knowledge and proceed to a higher level.
- Ongoing review and reinforcement of previously learned skills and concepts is of utmost importance.
- Giving learners good reasons for why we learn a topic and linking it to previous knowledge can help to remove barriers that stop a child from learning.
- Similarly, making an effort to explain where anything taught may be used in the future is also beneficial to the learning process.

How can this approach be implemented practically?

If there are a few learners in your class who are not grasping a concept, as a teacher, you need to find the time to take them aside and teach them the concept again (perhaps at a break or after school).

If the entire class are battling with a concept, it will need to be taught again. This could cause difficulties when trying to keep on track and complete the curriculum in the time stipulated.

Some topics have a more generous time allocation in order to incorporate investigative work by the learners themselves. Although this is an excellent way to assist learners to form a deeper understanding of a concept, it could also be an opportunity to catch up on any time missed due to remediating and re-teaching of a previous topic. With careful planning, it should be possible to finish the year's work as required.

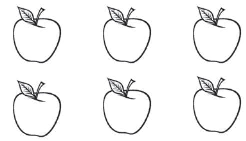
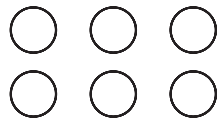
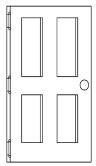
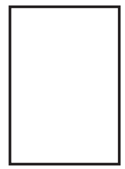
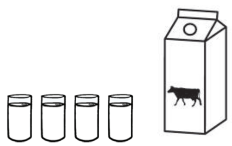
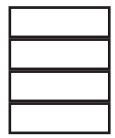
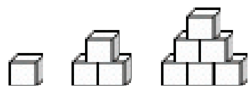
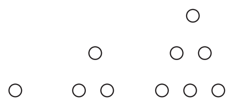
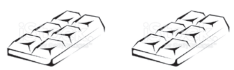
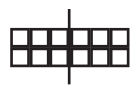
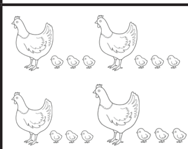
Another way to try and save some time when preparing for a new topic, is to give out some revision work to learners prior to the start of the topic. They could be required to do this over the course of a week or two leading up to the start of the new topic. For example, in Grade 8, while you are teaching the Theorem of Pythagoras, the learners could be given a homework worksheet on Area and Perimeter at Grade 7 level. This will allow them to revise the skills that are required for the Grade 8 approach to the topic.

What does this look like in the booklet?

At the beginning of each topic, there will be a SEQUENTIAL TEACHING TABLE, that details:

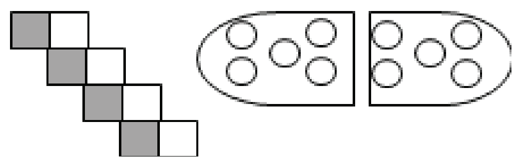
- The knowledge and skills that will be covered in this grade
- The relevant knowledge and skills that were covered in the previous grade or phase (looking back)
- The future knowledge and skills that will be developed in the next grade or phase (looking forward)

THREE-STEP APPROACH TO MATHEMATICS TEACHING: CONCRETE-REPRESENTATIONAL-ABSTRACT

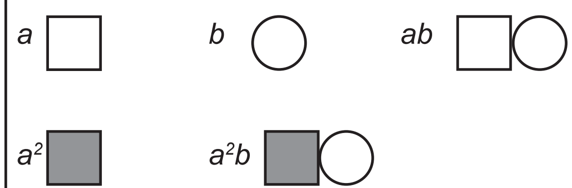
CONCRETE: IT IS THE REAL THING		REPRESENTATIONAL: IT LOOKS LIKE THE REAL THING		ABSTRACT: IT IS A SYMBOL FOR THE REAL THING	
Mathematical topic	**Real or physical For example:**	**Picture**	**Diagram**	**Number (with or without unit)**	**Calculation or operation, general form, rule, formulae**
Counting	Physical objects like apples that can be held and moved			6 apples	$2 \times 3 = 6$ or $2+2+2=6$ or $\frac{1}{2}$ of $6 = 3$ or $\frac{2}{3}$ of $6 = 4$
Length or distance	The door of the classroom that can be measured physically			80 cm wide 195 cm high	Perimeter: $2L + 2W = 390 + 160 = 550cm$ Area: $L \times W = 195 \times 80 = 15\,600cm^2 = 1.56m^2$
Capacity	A box with milk that can be poured into glasses			1 litre box 250 ml glass	$4 \times 250ml = 1\,000ml = 1$ litre or 1 litre $\div 4 = 0.25$ litre
Patterns	Building blocks			1; 3; 6...	$\frac{n(n+1)}{2}$
Fraction	Chocolate bar			$\frac{6}{12}$	$\frac{6}{12} = \frac{1}{2}$ or $\frac{1}{2}$ of $12 = 6$
Ratio	Hens and chickens			4:12	4 : 12 = 1 : 3 Of 52 fowls ¼ are hens and ¾ are chickens, ie 13 hens, 39 chickens
Mass	A block of margarine			500g	500g = 0,5 kg or calculations like 2 ½ blocks = 1,25kg

Teaching progresses from concrete -> to -> abstract. In case of problems, we fall back <- to diagrams, pictures, physically.

MODES OF PRESENTING MATHEMATICS WHEN WE TEACH AND BUILD UP NEW CONCEPTS

Examples	SPEAK IT: to explain the concept	SHOW IT: to embody the idea	SYMBOL IT: to enable mathematising
	• Essential for introducing terminology in context • Supports learning through the auditory pathway • Important to link mathematics to everyday realities	• Essential to assist storing and retrieving concepts • Supports understanding through the visual pathway • Important to condense a variety of information into a single image	• Essential to assist mathematical thinking about concepts • Supports the transition from situations to mathematics • Important to expedite calculation and problem solving
FP: Doubling and halving	"To double something, means that we make it twice as much or twice as many. If you got R50 for your birthday last year and this year you get double that amount, it means this year you got R100. If Mom is doubling the recipe for cupcakes and she used to use 2½ cups of flour, it means she has to use twice as much this time." "To halve something, means that we divide it into two equal parts or share it equally. If I have R16 and I use half of it, I use R8 and I am left with R8. If we share the 22 Astro's in the box equally between the two of us, you get eleven, which is one half and I get eleven, which is the other half."	1. Physical objects: Example: Double 5 beads Halve 12 beads 2. Pictures: Example: Double  Halve  3. Diagrams: 	7 + 7 = □ 7 + □ = 14 △ + ○ = 14, but □ + □ = 14 2 times 7 = 14 double of 7 is 14 14 – 7 = 14 – □ = 7 14 divided by 2 = 7 14 halved is 7

IP: Geometric patterns	"If we see one shape or a group of shapes that is growing or shrinking a number of times, every time in the same way, we can say it is forming a geometric pattern. If we can find out how the pattern is changing every time, we can say we found the rule of the sequence of shapes. When we start working with geometric patterns, we can describe the change in normal language. Later we see that it becomes easier to find the rule if there is a property in the shapes that we can count, so that we can give a number value to each , or each term of the sequence" "You will be asked to draw the next term of the pattern, or to say how the eleventh term of the pattern would look, for example. You may also be given a number value and you may be asked, which term of the pattern has this value?"	○ ○ ○○ ○ ○○ ○○○ Draw the next term in this pattern. T1 T2 T3 T4 ○ ○ ○○ ○ ○○ ○○○ ○ ○○ ○○○ ○○○○ Describe this pattern. What is the value of the 9th term of this pattern (T9)? ○ ○ ○○ ○ ○○ ○○○ ○ ○○ ○○○ ○○○○ ○ ○○ ○○○ ○○○○ ○○○○○ To draw up to the ninth term of this pattern, is a safe but slow way. It is even slower to find out by drawing, which term has a value of 120 for example. One is now almost forced to deal with this problem in a symbolic way.	Note how important it is to support the symbolising by saying it out: 1; 3; 6… 1; 3; 6; 10… 1; 3; 6;10;15 Inspecting the terms of the sequence in relation to their number values: T1: 1 = 1 The value of term 1 is 1 T2: 3 = 1+2 The value of term 2 is the sum of two consecutive numbers starting at 1 T3: 6 = 1+2+3 The value of term 3 is the sum of three consecutive numbers starting at 1 T4: 10 = 1+2+3+4 The value of term 4 is the sum of four consecutive numbers starting at 1 T5: 15 = 1+2+3+4+5 The value of term 5 is the sum of five consecutive numbers starting at 1 T9: 45 = 1+2+3+4+5+6+7+8+9 The value of term 9 is the sum of nine consecutive numbers starting at 1 We can see that the value of term n is the sum of n number of consecutive numbers, starting at 1.

SP: Grouping the terms of an algebraic expression	*"We can simplify an algebraic expression by grouping like terms together. We therefore have to know how to spot like terms. Let us say we have to sort fruit in a number of baskets and explain the variables or the unknowns in terms of fruits. Try to visualise the following pictures in your mind:"*	Although not in a real picture, a mind picture is painted, or a mental image to clarify the principle of classification: • Basket with green apples (a) • Basket with green pears (b) • Basket with green apples and green pears (ab) • Basket with yellow apples (a2) • Basket with yellow apples and green pears (a2b) Or in diagrammatic form 	$4b - a^2 + 3a^2b - 2ab - 3a + 4b + 5a - a - 2ab + 2a^2b + a^2b$ $= -3a + 5a - a + 4b + 4b - 2ab - 2ab - a^2 + 3a^2b + 2a^2b + a^2b$ $= a + 8b - 4ab - a^2 + 6a^2b$

TOPIC 1: FUNCTIONS AND RELATIONSHIPS

INTRODUCTION

- This unit runs for 5 hours.
- It is part of the Content Area, 'Patterns, Functions and Algebra', and counts for 35 % in the final exam.
- Functions and relationships were also done in Term 1. The focus in this term is on finding output values for given equations, and recognising equivalent forms between different descriptions of the same relationship.
- The purpose of this topic is to develop a firm understanding of the relationship between the input (x) and output (y) and to recognise that the output is unique for each specified input.
- It is important to note that the skills covered in Term 3 are much the same as in Term 1, but graphs will be introduced as another way of representing a function.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 8	GRADE 9	GRADE 10 / FET PHASE
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• Determine input values, output values or rules for patterns and relationships using flow diagrams, tables, formulae and equations • Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented verbally, in flow diagrams, in tables, by formulae and by equations	• Determine input values, output values or rules for patterns and relationships using flow diagrams, tables, formulae and equations • Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented verbally, in flow diagrams, in tables, by formulae, by equations by graphs on a Cartesian plane	• A more formal definition of functions will be explored • In Gr 10 learners will study parabolas (quadratic), hyperbolas and exponential graphs as well as trigonometric graphs. Vertical shifts (q) and shape and stretch (a) are studied. • In Gr 11, the horizontal shift is included • In Gr 12, inverse functions, including the logarithmic graph are included

GLOSSARY OF TERMS

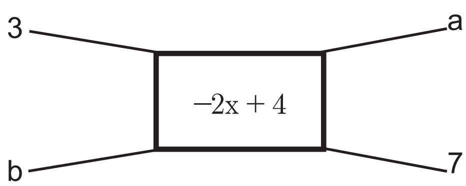
Term	Explanation / Description
Function	A mathematical condition or rule linking the input to the output.
Relation	A relation is a relationship between sets of values. In maths, the relation is between the x-values and y-values of ordered pairs
Input	The number/value that was chosen to replace the variable in an expression.
Output	The output is dependent on the input – it is the answer once the operation has been performed according to the expression given.
Equation	A mathematical sentence built from an algebraic expression using an equal sign. Example: 2a + 3 = 10
Expression	A mathematical model which represents a situation. It can include variables (letters), constants and operations. Example: 2b + 3c
Flow Diagram	A diagram representing a sequence of movements to be performed on a given value.
Algebraic Rule	An expression representing a rule to be performed on the variable. Example: 3m + 1 (Multiply the number represented by 'm' by 3 then add 1 to the answer)
Inverse Operation	The opposite operation that will 'undo' an operation that has been performed. Addition and subtraction are the inverse operation of each other. Multiplication and Division are the inverse operation of each other.

SUMMARY OF KEY CONCEPTS

As this topic was also covered in Term 1, some revision of the concepts from then should be done in the first lesson. Refer back to the Term 1 booklet for this.

Input and Output

1. To find output, substitution is used. To find input, solving of equations is used.
   For example:



To find 'a' (output): Replace x with 3 (use substitution)

$$=-2(3)+4$$
$$=-6+4$$
$$=-2 \qquad \therefore b=-2$$

Teaching Tip:
Remind learners of the importance of using brackets when substituting. This is a concept that should have been learnt in Grade 8.

To find 'b' (input): This requires working in reverse which in turn requires solving equations.
First, replace the x with 'b', making the expression equal to 7 and then solve for 'b'.

$$\begin{aligned} -2b+4 &= 7 \\ -2b &= 7-4 \\ -2b &= 3 \\ \frac{(-2b)}{-2} &= \frac{3}{-2} \\ b &= \frac{-3}{2} \end{aligned}$$

Teaching Tip:
Remind learners of the process to solve equations using inverse operations. In the above example, the '+4' in the first step is removed by subtracting 4 from both sides. The 'times -2' is removed by dividing by -2 on both sides.

2. Input and output can also be found when a table is given. It will be done in the same way as before. Substitution will be used to find the output when given the input and solving equations will be used to find input when given output.

   For example:

a	4	5	[3]	[4]
$\frac{a}{2}+3$	[1]	[2]	8	4

To find (1) and (2) involves finding the output. To find (3) and (4) involves finding the input.

(1) $\frac{4}{2}+3=2+3=5$

(2) $\frac{5}{2}+3=\frac{5+6}{2}=\frac{11}{2}$

(3)
$$\begin{aligned}\frac{a}{2}+3 &= 8\\ \frac{a}{2} &= 5\\ a &= 10\end{aligned}$$

(4)
$$\begin{aligned}\frac{a}{2}+3 &= 4\\ \frac{a}{2} &= 1\\ a &= 2\end{aligned}$$

Rules

1. When given both the input and the output, learners can be asked to find the rule.
2. To find the rule when only one operation has been performed to get from the input to the output was covered extensively in Term 1.

a	1	2	3	4
b	1	3	5	7

3. To find the rule when 2 operations have been performed to get from the input to the output was covered briefly but now needs to be consolidated.

Steps to follow to find rule:

- Look at the output numbers and find the common difference (in this case 2)
- This tells us that the first part of the rule is 'times 2'.
- Look at the input numbers and times them by 2. What still needs to be done to get to the output? (in this case subtract 1)

- Therefore, the rule is: 'times 2, subtract 1'.
- Rule: a x 2 - 1 = b, which is better written as: 2a - 1 = b

x	1	2	3	4
y	0	-2	-4	-6

The common difference of the output values: - 2
($\therefore$ first part of the rule is: multiply by - 2)
Input numbers x -2: - 2 ; - 4 ; - 6 ; - 8
Operation still needed to get from these values to output values: + 2

$\therefore \times -2$ and $+2$ (multiply by -2 and add 2)
$x \times -2 + 2$
$-2x + 2 = y$

Equivalent forms

1. There are a number of ways that a relationship between two variables can be expressed. It can be described in words, shown in a flow diagram, shown in a table, as an equation or formula and it can also be represented visually as a graph. It is important to point out to learners that they need to be able to change from one representation to another. For example, when given a graph, they need to be able to make a table or when given an equation, they need to be able to draw a graph.

Graph ⟷ table ⟷ equation ⟷ flow diagram

2. When representing the relationship as a graph, it is usually linked to representing it as an equation or in table form beforehand.

For example: A relationship is given as y=2x-4
This equation can be used to complete the following table, using substitution:

x	-2	-1	0	1	2
y					

The 5 missing values should be: -8 ; - 6 ; - 4 ; - 2 ; 0
The co-ordinates formed from the input (x) and the output (y) values can be plotted on a Cartesian Plane.

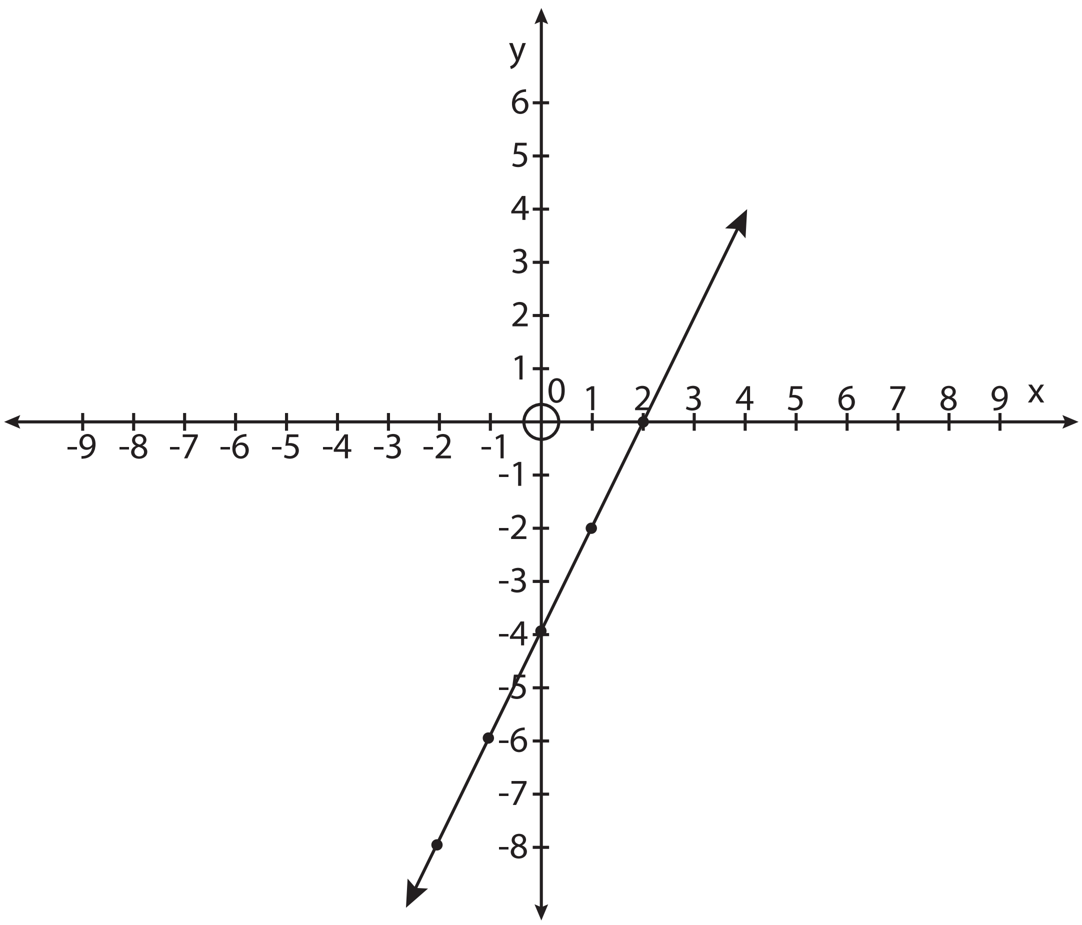
Teaching Tip:

Learners may need reminding how to plot points on the Cartesian plane as it was covered in Grade 8 and was also new to learners then. When Graphs are done later in the term this will need to be revised again. It is recommended at this time not to spend too much time making the learners draw their own Cartesian planes as that is not the focus of this section. If possible, learners can be supplied with one or two to draw a few graphs on. There is a sheet available in the resource section that can be used for photocopying purposes if school resources allow.

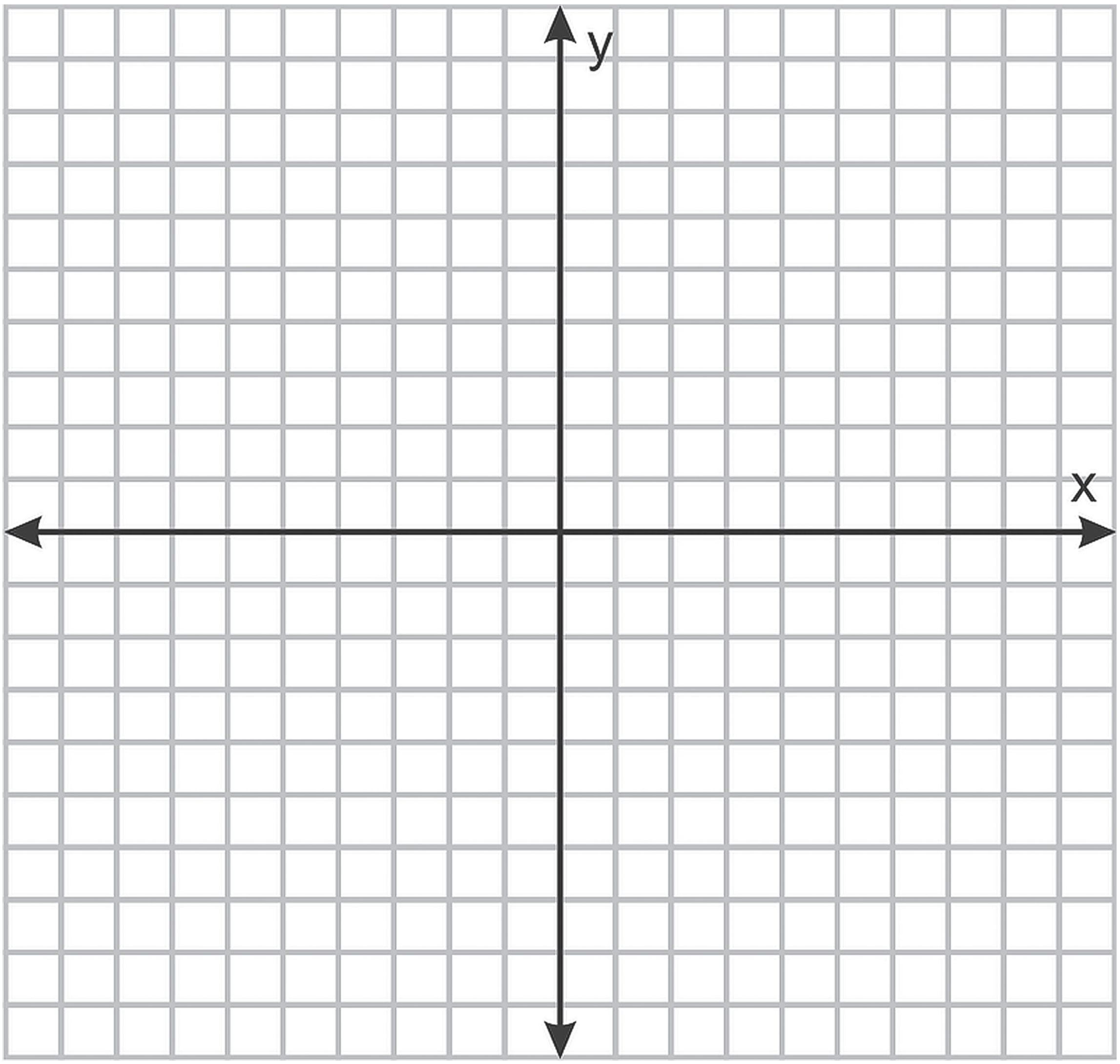
Points to be plotted to represent the relationship $y = 2x - 4$ as a graph:

(-2 ; -8) ; (-1 ; -6) ; (0 ; -4) ; (1 ; -2) ; (2 ; 0)

This is a good opportunity to remind learners of flow diagrams. The input and output of a flow diagram are represented here as the 1st value of the co-ordinate and the 2nd value of the co-ordinate



RESOURCES



TOPIC 2: ALGEBRAIC EXPRESSIONS

INTRODUCTION

- This unit runs for 9 hours.
- It is part of the Content Area, 'Patterns, Functions and Algebra', which counts for 35 % in the final exam.
- The unit this term has a focus on the use of exponential laws and factorisation.
- It is important to note that this topic was started in Term 1. Learners should already know how to multiply a binomial with a binomial and square a binomial.
- The purpose of this section is to form the foundation for further studies in Mathematical Sciences.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 8	GRADE 9	GRADE 10 / FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Identify variables and constants • Recognize conventions for writing algebraic expressions • Identify and classify like and unlike terms • Recognize and identify coefficients and exponents • Add and subtract like terms • Multiply monomials by monomials, binomials, trinomials • Divide monomials, binomials and trinomials by integers or monomials	• Determine the squares, cubes, square roots and cube roots of algebraic terms • Find the product of two binomials and square a binomial • Factorise algebraic expressions involving common factors, difference of two squares and trinomials of the form: $x^2 + bx + c$ and $ax^2 + bx + c$ where a is a common factor. • Simplify algebraic fractions using factorisation	• Apply laws of exponents to expressions involving rational exponents • Work with surds • Use factorisation to solve quadratic equations • Solve quadratic inequalities • Solve simultaneous equations (including quadratic) algebraically and graphically

GLOSSARY OF TERMS

Term	Explanation / Description
Expression	A mathematical model which represents a situation. It can include variables (letters), constants and operations. Example: 2b + 3c
Product	The answer to a multiplication question.
Sum	The answer to an addition question.
Difference	The answer to a subtraction question.
Quotient	The answer to a division question.
Term	Part of an algebraic expression. Terms are separated by '+' or '-' signs.
Coefficient	A number or symbol (including its sign) multiplied with a variable in a term. For example, 2 is the coefficient of y in the term 2y 2b is the coefficient of a in the term 2ab
Variable	Letters of the alphabet which could represent different values Example: In the expression m + 2, m is a variable and could be replaced by a number in order to calculate the answer when m is equal to that specific number. Variables can change values.
Constant	A number making up a term on its own in an expression Example: a + 3b – 10: -10 is the constant. Constants cannot change value.
Substitution	Replacing a variable (letter of the alphabet) with a number to perform a calculation. Example: If b = 3 , then b + 2 = 3 + 2 = 5
Monomial	One term expression.
Binomial	Two term expression.
Trinomial	Three term expression.
Polynomial	More than one term expression (two or more).
Like Term	Terms that have exactly the same variables. Example: 2a and 4a are like terms and can be added or subtracted 3abc and 10abc are like terms and can be added or subtracted
Unlike term	Terms that do not have the same variables. Example: 3a and 2b are unlike terms and cannot be added or subtracted x and y are unlike terms and cannot be added or subtracted
Exponent	In the example a^2 , '2' (or squared) is the exponent. It is the number or variable written at the top of the base in smaller font.
FOIL	Acronym short for: First, Outer, Inner, Last. A method for multiplying a binomial by a binomial.

SUMMARY OF KEY CONCEPTS

Expanding and simplifying algebraic expressions

1. Remember: We can ONLY add and subtract LIKE terms;

   The 'names' of terms don't change, only the coefficient does (AND the sign to the left of a term belongs to that term!)

   For example:
   a. $5x + 2x = 7x$ (new coefficient but x remains the same)
   b. $6y - 2y = 4y$
   c. $7x + 4y = 7x + 4y$ (unlike terms so can't add or subtract)
   d. $5ab - 4a + 2a + 3ab = 8ab - 2a$

   Note: It is worth keeping it in mind (and pointing out to learners if possible) that in a statement such as $5x + 2x = 7x$, not only are we adding like terms but in fact we are also dealing with an equation that is stating that this statement is true for all values of x.

Teaching Tip:
Where possible try and use real life examples when practicing the addition and subtraction of like terms. For example, ask learners what 3 pens + 4 pens is. They will probably answer correctly, i.e.: 7 pens. ($3p + 4p \neq 7p2$) is a common error when learners first start Algebra, but if you say 3 books + 7 books, they are unlikely to make this mistake). Using real life examples should help them to see that a 'name' doesn't change when adding and subtracting like terms.

Teaching Tip:
Using the same idea you can also show that unlike terms cannot be added or subtracted. For example, ask learners what 3 pens + 4 cars is. They should see that this would have to remain 3 pens + 4 cars and will therefore have a better understanding that they cannot add and subtract terms unless they are exactly the same.

2. Simplification of expressions involving brackets:

   Brackets show that multiplication needs to take place. If the terms inside the brackets are unlike, the DISTRIBUTIVE LAW is required.

For example:

a. $3(a + b)$ (the 3 needs to multiplied by both terms inside the bracket)
$= 3a + 3b$
Integer and exponent rules always apply!

b. $-2(m - n)$
$= -2m + 2n$

c. $3a(a + b) - 2(a^2 - ab)$
$= 3a^2 + 3ab - 2a^2 + 2ab$
$= a^2 + 5ab$

d. $-x^3(-x + xy)$
$= x^4 - x^4y$

3. Division of a polynomial by a monomial

When more than one term is divided by only one term, each term in the dividend (numerator) needs to be divided by the divisor (denominator)

Examples:

$\frac{10x^2 - 5x + 5}{x}$ $= \frac{10x^2}{x} - \frac{5x}{x} + \frac{5}{x}$ $= 10x - 5 + \frac{5}{x}$	$\frac{4x + 8}{2} + \frac{5x - 3}{5}$ $= \frac{4x}{2} + \frac{8}{2} + \frac{5x}{5} - \frac{3}{5}$ $= 2x + 4 + x - \frac{3}{5}$ $= 3x + \frac{17}{5}$

4. Simplification of algebraic expressions using exponential laws

$a^m \times a^n = a^{m+n}$	$a^m \div a^n = a^{m-n}$
$(a^m)^n = a^{m \times n} = a^{mn}$	$(ab)^n = a^n b^n$

For example:

$(x^2y)^3$ $= x^6y^3$	$2a^2b(a^3b + ab^2)$ $= 2a^5b^2 + 2a^3b^3$	$\frac{20m^4n^2p}{-5m^2n}$ $= -4m^2np$

5. Substitution

Because the letters represent variables and can take on many values we need to be able to substitute these variables for certain values.

Examples:

1. Find $5a + 2b$ if $a = 3$ and $b = 2$

$$\begin{aligned} 5a + 2b &= 5\,(3) + 2\,(2) \\ &= 15 + 4 \\ &= 19 \end{aligned}$$

(note the use of brackets instead of times signs)

2. Find $3x - y + 2v$ if $x = -1$ $y = -2$ $v = -3$

$$\begin{aligned} 3x - y + 2v &= 3\,(-1) - (-2) + 2\,(-3) \\ &= -3 + 2 - 6 \\ &= -7 \end{aligned}$$

3. Find $4x^2 - y^3$ if $x = -1$ $y = 2$

$$\begin{aligned} 4x^2 - y^3 &= 4(-1)^2 - (2)^3 \\ &= 4\,(1) - 8 \\ &= 4 - 8 \\ &= -4 \end{aligned}$$

NOTE: Learners need to be proficient in their integer and exponent work in order to cope with substitution.

Teaching tip:

Experience has shown that learners not only find substitution easier if they follow the steps shown below but also make fewer errors:

- Rewrite the expression given, but every time a variable appears, open and close a bracket
- Now check what value has been allocated to each variable and put it inside the empty bracket in the correct variables place.
- Use BODMAS to find the value of the expression

For example:

If $p = 2$, $q = -2$ and $r = 3$ find the value of $2p^2 q + (qr)^2$

STEP 1: $2(\ \ )^2(\ \ )+((\ \ )(\ \ ))^2$

STEP 2: $2(\,2\,)^2(-2\,)+((-2\,)(\,3\,))^2$

STEP 3: $2(4)(-2)+(-6)^2$

$= -16+36$

$= 20$

(NOTE: Step 1 and 2 would not show as 2 steps when the learners have completed the question, as step 2 would have been done as soon as step 1 was complete)

Squares, square roots, cubes and cube roots

1. Learners should already be familiar with squaring, cubing, square rooting and cube rooting from Grade 8. The extra skill needed at this stage is to extend this knowledge to algebraic expressions.

Teaching tip:
It is advisable again to encourage learners to find the root (square or cube) of each base in the expression one at a time as opposed to trying to look at the question as one 'big' thing to do and struggling with it.

For example:
$\sqrt{(9x^2)}$: The focus should be on finding the square root of 9 and then finding the square root of x^2

$(5x^2)^2$: The focus here should be on first squaring the 5 and then squaring the x^2.

$\sqrt{(9x^2)} = 3x$
$(5x^2)^2 = 25x^4$

If a negative integer is involved, remind learners to focus on that first before moving on to the other bases.

For example: $(-3a^2b^5)^2 = 9a^4b^{10}$

Steps followed to find this solution:
First square the negative, then square the 3, then use the exponential laws to square the a^2 and then the b^5.

2. Examples to expect at Grade 9 level:

$\sqrt{54a^6b^{10} + 10a^6b^{10}}$
$= \sqrt{64a^6b^{10}}$
$= 8a^3b^5$

(The root sign needs to be treated like a bracket and any operations to be performed inside need to be done first)

The product of two binomials and squaring a binomial is covered in the Term 1 book.

To factorise is to perform the opposite to expanding and simplifying. The aim of factorising is to reduce an expression from more than one term to one term only.

Factorising - Common factors

1. In order to find a common factor in an expression, learners need to be finding the highest common factor. This was covered in Grade 8.
2. When looking for a common factor in an algebraic expression it is important that a learner can see how many terms he/she is dealing with. This was covered in Grade 8 algebra and may need revisiting.
3. Once the number of terms has been established, the HCF (Highest Common factor) of the terms needs to be found. This is what will be taken out of the expression.

   For example:
   In the expression, $10a + 20$, there are two terms. The HCF is 10.
   If 10 is taken out of the expression, it needs to be established what needs to be multiplied by 10 to get the original expression back if the distributive law were to be applied.

   $10a + 20 = 10\ (a + 2)$

   Note that $10 \times a = 10a$ (the first term of the expression) and that $10 \times 2 = 20$ (the second term of the expression) In order to find the terms that remain inside the bracket, division of a monomial by a monomial is used. In this case, $10a \div 10$ and $20 \div 10$

4. Further examples involving more complicated expressions involving exponents:

Teaching Tip:

Note and point out to learners the following in each example:

- The number of terms in the original expression is always the same as the number of terms 'left over' inside the bracket
- The final answer is always one term, unless no factorisation was possible then the answer will look the same as the original question

$2a^2 - 2ab$

$= 2a(a - b)$

These 2 terms:

$2a^2 = 2 \times a \times a$

$2ab = 2 \times a \times b$

$2a$ is common

$(a - b)$ in the remainders bracket

$3a^3 - 6a^2 + 18ab$

$= 3a(a^2 - 2a + 6b)$

These 3 terms:

$3a^3 = a \times a \times a$

$6a^2 = 3 \times 2 \times a \times a$

$18ab = 3 \times 3 \times 2 \times a \times b$

$3a$ is common

$(a^2 - 2a + 6b)$ in the remainders bracket

$-3a^3 - 6a^2 - 6ab$

$= -a(3a^2 + 6a + b)$

When the first term is a negative, always take the negative out as part of the common factor. This will change all the signs of the original expression when finding the 'leftover' terms inside the bracket

5. Common factors can also be found in more complex expressions

   Consider the following expression: $3x(a + 1) - 2(a + 1)$

   Firstly, it is important to note that there are two terms in the expression. The common factor to both of these terms is (a+1). Once this has been taken out as the HCF, it needs to be established what that factor will need to multiplied with to find what the new terms in the bracket will be.

   Solution: $3x(a + 1) - 2(a + 1)$

   $= (a + 1)(3x - 2)$

   Division done to find the 2nd bracket terms: $\frac{3x(a+1)}{(a+1)}$ and $\frac{-2(a+1)}{(a+1)}$

This idea is extended in the following situation:
There are times when we need to group terms in order to make factorising easier. When we group the terms it looks similar to the previous example and we can find a common factor which will be a common bracket. If there are four terms and no common factor can be found in all 4 terms, it needs to be checked if grouping can result in a common factor and then common remainder brackets. In the following example, note that if we consider the first two terms, a common factor can be found.

Then the second two terms can be considered on their own where a common factor can also be found. The important thing to remember is that we require the leftover bracket to be a new common factor in the two terms which are the result from factorising the 4 terms in pairs.

$$2x + 2y + kx + ky$$
$$= 2(x + y) + k(x + y)$$
$$= (x + y)(2 + k)$$

It is important to note that the above example could also have been completed in the following way:

$$(2x + kx) + (2y + ky)$$
$$= x(2 + k) + y(2 + k)$$
$$= (2 + k)(x + y)$$

The commutative property can let us claim that:

$$(x + y)(2 + k) = (2 + k)(x + y)$$

A further example

$$x^2 + xy + xt + yt$$
$$= x(x + y) + t(x + y)$$
$$= (x + y)(x + t)$$

Factorising – Difference of two squares

1. If the following expression is multiplied out and simplified it forms a certain type of binomial.

$(x-2)(x+2)$ (notice that the terms in the brackets are identical except for the signs)

$= x^2+2x-2x-4$

$= x^2-4$ (note that the expression consists of two terms only with a minus sign between them and that both terms are perfect squares)

Teaching Tip:
Spend some time asking learners to test if this always happens when multiplying a binomial by a binomial that have identical terms but different signs.

Use the following as examples for them to expand and simplify:

$(a+3)(a-3)$; $(2y-1)(2y+1)$; $(3x^2+4)(3x^2-4)$; $(10+ab)(10-ab)$

2. The solution to all of the above is called the difference of two squares (two perfect square numbers with a subtraction sign between them)
In order to factorise a difference of two squares:
   - Open two brackets (to represent the two factors)
   - Place an addition sign in one bracket and a subtraction sign in the other bracket
   - Square root term one and place the answer in the front of each bracket
   - Square root term two and place the answer at the back of each bracket

For example:
Factorise: a) $d^2-49=(d+7)(d-7)$
b) $a^4b^2-100c^2=(a^2b+10c)(a^2b-10c)$

These questions can also be combined with finding a common factor first.
For example:

$2x^8 - 162$	Note that 2 is a common factor to both terms
$= 2(x^8 - 81)$	Note that the two terms left over in the bracket form a difference of two squares
$= 2(x^2 + 9)(x^2 - 9)$	Note that the second bracket is still a difference of two squares
$= 2(x^2 + 9)(x + 3)(x - 3)$	

It is important to point out to learners that a SUM of two squares CANNOT be factorised.

Factorising – Trinomials

Teaching Tip:
When starting this section, it is a good idea to have a look back at multiplying a binomial with a binomial. This way learners can see that a trinomial is the result. This should help them see that the trinomial can in turn be factorised into one term comprising of two factors (shown by the two brackets). Remind learners that factorising is the inverse of multiplying out and simplifying.
Examples similar to the following can be used:

$(a + 3)(a + 2)$; $(y - 1)(y - 2)$; $(x - 1)(x + 4)$; $(x + 5)(x - 2)$

Note that each type which will be discussed below is covered in these 4 examples.

1. When a trinomial has a coefficient of 1 for the x^2, there are two hints that will help:
   - When the last sign is addition, both signs are the same and match the middle term.
   - When the last sign is subtraction, both signs are different and the larger number goes with the sign of the middle term.

2. Consider the trinomial $x^2 + 7x + 10$
   In order to factorise this trinomial:
   - Open two brackets (which represent the two factors)
   - Use the above rules to decide on the signs (in this case it will be + and + in each bracket)
   - Square root the first term and put the solution at the front of each bracket
   - Because the signs ARE THE SAME, ask, 'what two numbers multiply to make the last term but also ADD UP to make the middle term' (in this case 5 and 2)
   - Place these two factors at the back of each bracket. As the signs are the same it doesn't matter which number goes into each bracket
     Solution: $x^2 + 7x + 10 = (x + 5)(x + 2)$

3. Examples of the other types:
   a. $x^2 - 7x + 12$
   - Open two brackets (which represent the two factors)
   - Use the above rules to decide on the signs (in this case it will be - and - in each bracket)
   - Square root the first term and put the solution at the front of each bracket
   - Because the signs ARE THE SAME, (even though they are subtraction signs) ask, 'what two numbers multiply to make the last term but also ADD UP to make the middle term' (in this case 4 and 3)
   - Place these two factors at the back of each bracket. As the signs are the same it doesn't matter which number goes into each bracket
     Solution: $x^2 - 7x + 12 = (x - 4)(x - 3)$

b. $x^2 - 2x - 15$

- Open two brackets (which represent the two factors)
- Use the above rules to decide on the signs (in this case it will be - and + in each bracket. At this stage each one can be placed in any bracket)
- Square root the first term and put the solution at the front of each bracket
- Because the signs ARE DIFFERENT, ask, ‘what two numbers multiply to make the last term but also SUBTRACT to make the middle term’ (in this case 5 and 3)
- As the signs are different, it will matter where the numbers are placed. The biggest number ALWAYS goes with the sign in front of the middle term (in this case the 5 will go with the ‘-‘)
  Solution: $x^2 - 2x - 15 = (x - 5)(x + 3)$

c. $x^2 + 7x - 18$

- Open two brackets (which represent the two factors)
- Use the above rules to decide on the signs (in this case it will be - and + in each bracket. At this stage each one can be placed in any bracket)
- Square root the first term and put the solution at the front of each bracket
- Because the signs ARE DIFFERENT, ask, ‘what two numbers multiply to make the last term but also SUBTRACT to make the middle term’ (in this case 9 and 2)
- As the signs are different, it will matter where the numbers are placed. The biggest number ALWAYS goes with the sign in front of the middle term (in this case the 9 will go with the ‘+‘)

Solution: $x^2 + 7x - 18 = (x + 9)(x - 2)$

Teaching Tip:
Learners struggle with this section in the beginning. It is essential that they practice as many examples as possible. It is also recommended that each type be first practiced separately in order for learners to gain more confidence before moving on to all types of trinomials being combined where each one needs to be thought about before the factorising can commence.

Factorising – Fractions

1. When dividing algebraic expressions that have more than one term in the numerator and denominator, factorisation is often essential in order to be able to simplify.
2. Learners need to be reminded that no term next to a '+' or '–' can be simplified in isolation.

   For example: $\frac{2a+5}{a} \neq 2+5$

   (The 'a' in the denominator cannot be divided into the 'a' in the numerator.)

3. Consider the following fraction: $\frac{5x^2-10x}{x-2}$

   In order to simplify this fraction, the numerator first needs to be factorised. There is a common factor of $5x$ in the two terms that form the numerator and can therefore be taken out.

   $$\frac{5x^2-10x}{x-2} = \frac{5x(x-2)}{x-2}$$

   The $(x-2)$ in the denominator can now be divided into the $(x-2)$ in the numerator.

   $$\frac{5x^2-10x}{x-2} = \frac{5x(x-2)}{x-2} = 5x$$

TOPIC 3: ALGEBRAIC EQUATIONS

INTRODUCTION

- This unit runs for 9 hours.
- It is part of the Content Area, 'Patterns, Functions and Algebra', and counts for 35 % in the final exam.
- The unit covers all previous equations done in Grade 8 and earlier in the year in Grade 9 and goes on to include solving equations using factorisation.
- It is important to note that equations were done in Term 1. The focus in this term is on solving equations using factorisation, and equations with a product of factors. This term also focuses on using equations to generate tables of ordered pairs.
- The purpose of learning to solve Equations is also to help with Problem Solving which is the basis of all mathematics.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 8	GRADE 9	GRADE 10/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Solve equations by inspection • Determine the numerical value of an expression by substitution. • Identify variables and constants in given formulae or equations • Use substitution in equations to generate tables of ordered pairs • Extend solving equations to include using additive and multiplicative inverses and using laws of exponents	• Set up equations to describe problem situations • Solve equations by inspection using additive and multiplicative inverses and the laws of exponents • Determine the numerical value of an expression by substitution. • Use substitution in equations to generate tables of ordered pairs • Solve equations to include the use of factorisation and equations of the form where a product of factors = 0	• Quadratic equations are a main focus in the FET phase. particularly in Grade 11 and 12 • Inequalities (including quadratic) are covered in more depth in Grade 11 • Exponential equations • Logarithmic equations

GLOSSARY OF TERMS

Term	Explanation / Description
Equation	A mathematical statement with an equal sign that includes a variable For example: $3x - 5 = 20$
Expression	An algebraic statement consisting of terms with variables and/or constants. There is no equal sign For example: $3a + 2b$ or $3 + 2$ or $a + b$
Formula	A formula is used to calculate a specific type of answer and has variables that represent a certain kind of value For example: Area = $l \times b$. This formula finds area of a rectangle and only measurements can replace the l and b
Variable	Letters of the alphabet which could represent different values For example: In the expression $m + 2$, m is a variable and could be replaced by a number in order to calculate the answer when m is equal to that specific number. Variables can change values
Like Terms	Terms that have exactly the same variables For example: $2a$ and $4a$ are like terms and can be added or subtracted
Inverse Operation	The opposite operation that will 'undo' an operation that has been performed. Addition and subtraction are the inverse operation of each other. Multiplication and Division are the inverse operation of each other.
Solution	The value of the variable that makes the two sides of the equation balance.
Identity	An equation that is true for any values that replace x. That means x can be anything. $x \in R$
Distributive law	This indicates how we share operators that are linked together. For example: $c(a + b)$ $= c\ (a + b)$ $= (c \times a)\ (c \times b)$ $= ca + cb$
Equate	To put the LHS (left hand side) term(s) equal to the RHS right hand side term(s)

SUMMARY OF KEY CONCEPTS

The following was covered earlier in the year and can be found in the Term 1 book:
Solving equations by:

- inspection
- using additive and multiplicative inverses
- equations involving the product of factors (this will be extended in this term)

Solving equations using factorisation

1. In the previous topic, learners factorised algebraic expressions. This can now be used to solve quadratic equations. Quadratic equations are those that have a variable squared. For example: $x^2=25$ the exponent (2-squared) shows that there are TWO solutions to this equation.
   While working with the theorem of Pythagoras learners were taught to square root both sides of such an equation which gave them an answer of 5. The reason this was allowed is because the theorem deals with lengths of sides and it is impossible for a length or a distance to be anything other than positive. However, there is in fact one other number that can be squared to give 25 and that is '-5'
2. In order to solve quadratic equations, these steps need to be followed:
   - Ensure all terms are on one side (usually the left hand side) of the equal sign and zero on the other
   - Factorise the expression on the LHS (left hand side)
   - The factorised expression should be made up of two factors
   - If two factors multiply to make zero then either one of these factors needs to equal zero (any number multiplied by zero always equals zero)
   - State the two options, making each factor equal to zero and solve each equation.
   - For example:

a)

$$\begin{aligned} x^2 &= 25 \\ x^2 \quad 25 &= 0 \\ (x+5)(x-5) &= 0 \\ \therefore Either\ \ x+5 &= 0 \qquad or\ \ x-5 = 0 \\ \therefore x &= -5 \qquad or\ \ x = 5 \end{aligned}$$

b)

$$
\begin{aligned}
x^2 - 2x &= 0 \\
x(x-2) &= 0 \\
\text{Either } x = 0 \quad & or \quad x - 2 = 0 \\
& \qquad x = 2
\end{aligned}
$$

c)

$$
\begin{aligned}
x^2 + 5x - 6 &= 0 \\
(x+6)(x-1) &= 0 \\
\text{Either } x + 6 = 0 \quad & or \quad x - 1 = 0 \\
x = -6 \quad & or \quad x = 1
\end{aligned}
$$

Teaching Tip:
Point out to learners that in order solve this type of equation the aim is to have two factors multiplied to equal zero. Questions in textbooks and tests are often asked and already in that format and learners tend to multiply out then try to solve. For example: $(x+1)(x-4)=0$ is already in the format required so there is no need to multiply out. The next step will be to split into the two equations and solve.

Use substitution in equations to generate tables of ordered pairs

1. Substitution has been used a few times in topics prior to this one. It is used here again involving equations and will be used again in the following topic, graphs.
2. For example:
   Complete the table using the given equation to fill in the missing values:

x	-2	-1	0	1	2
$y = 2x + 3$					

Solutions

$y = 2x+3$ $y = 2(-2)+3$ $y = -4+3$ $y = -1$	$y = 2x+3$ $y = 2(-1)+3$ $y = -2+3$ $y = 1$	$y = 2x+3$ $y = 2(0)+3$ $y = 0+3$ $y = 3$	$y = 2x+3$ $y = 2(1)+3$ $y = 2+3$ $y = 5$	$y = 2x+3$ $y = 2(2)+3$ $y = 4+3$ $y = 7$

Teaching Tip:
Learners should find this fairly easy now that they have done this in a few topics as well as in Grade 8. If time permits, it would be a good idea to link this back to number patterns.

Solving word problems

(Setting up equations to describe problem situations)

The final purpose of all equations is to be able to model word problems (story sums) into equations that can be solved to find the answer to the problem.
Guidelines for solving word problems:

1. Read each problem three times. The first reading is to determine what the problem is, the second to identify the "maths" words and the third is to set up and equation.
2. Decide what exactly is being asked and make this the variable ready to be represented in the equation. If there are two unknowns make the smaller one the variable chosen.
3. If two or more items are involved express one in terms of the other. For example: The boy is twice as old as his sister. The sister is younger so she can be represented by the variable chosen (x being the most common) and the boy's age will then be $2x$
4. Set up your equation using any other information given in the statement.
5. Solve the equation.
6. Answer the question asked.
7. Make sure the answer makes sense. For example, age cannot be negative

There are different type of word problems:

Type 1- Numbers

x is a natural number therefore the next two numbers would be
$x + 1$ and $x + 2$
This is useful in a question such as, The sum of two consecutive numbers is 83. Find the numbers.
Solution: Let the first number be x.
Therefore the second number will be $x + 1$

These two numbers must add together to make 83
$\therefore x + x + 1 = 83$

Solve the equation: $x + x + 1 = 83$

$$\begin{aligned} 2x + 1 &= 83 \\ 2x &= 82 \\ x &= 41 \end{aligned}$$

Answer the question: The two numbers are 41 and 42

Type 2 – Age

A table is always useful in these types of questions. There will always be some information about now and other information about some time in the past or the future. Now and the other time mentioned will be the columns in the heading and the two people involved will be the headings for the rows. Call the youngest person the variable.
For example: A father is now 3 times as old as his son. Eight years ago their combined age was 64 years. How old is the father now?

Person	Now	8 years ago (means minus)
Father	$3x$	$3x - 8$
Son	x	$x - 8$

Use the information underneath the time other than now heading to form an equation with what was told in the question. (In this case, the two ages added together is equal to 64)

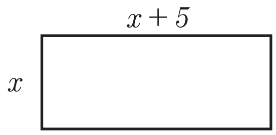
$$\begin{aligned} x - 8 + 3x &= 64 \\ 4x - 16 &= 64 \\ 4x &= 80 \\ x &= 20 \end{aligned}$$

That means the son is 20 and the father is 60 years old.

Type 3 – Perimeter and Area

Teaching Tip:
It is often useful to make a sketch for these questions.
For example: The length of a rectangle is 5cm more than the width. The perimeter is 160cm. Find the length.

Let the width be x and the length be x + 5



$$P = 2(l + w)$$
$$160 = 2(x + x + 5)$$
$$80 = 2x + 5$$
$$75 = 2x$$
$$x = 37{,}5cm$$

The width is 37,5 cm and the length is 42,5 cm

Type 4 – Money

For example: An amount of R34,80 is made up of 50c and 20c coins. How many 50c coins are there out of a total of 120 coins?

No of coins	Value of coins	Total value
x	50c	$50x$
$120 - x$	20c	$2400 - 20x$

$$\begin{aligned} 50x + 2400 - 20x &= 3480 \\ 30x &= 1080 \\ x &= 36 \end{aligned}$$

There are 36 coins that are 50c coins.

Type 5 – Speed, Distance and Time

Remember: Distance = speed x time

For example: A certain distance is covered in 3 hours at 72km/h. How long will the same journey take at 96km/h?

$D = 3 \times 72$

$D = 216km$

$t = \frac{D}{s}$

$t = \frac{216}{96}$

$t = 2,25$ hours

2 hours $0,25 \times 60 = 15\text{min}$

The trip will take 2 hours and 15 minutes.

Note: The above 5 types of questions do not cover every possible problem. They do however cover most of what tends to be asked at this level.

TOPIC 4: GRAPHS

INTRODUCTION

- This unit runs for 12 hours.
- It is part of the Content Area, 'Patterns, Functions and Algebra', and counts for 35% in the final exam.
- The unit covers analysing graphs as well as drawing the straight line graph and finding its equation from given information.
- It is important to note that although this work is new to learners there are many skills that have been covered previously which will help immensely in this section. These include solving equations and substitution.
- Graphs are visual representations of numerical systems and equations.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 8	GRADE 9	GRADE 10 / FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Interpreting and drawing graphs • Understanding of maximum and minimum • Understanding of discrete and continuous • Use tables/ordered pairs to plot points and draw graphs on the Cartesian plane • Equivalent forms	• Interpreting and drawing graphs • Understanding of maximum and minimum • Find x and y intercepts of linear graphs • Find gradient of linear graphs • Draw linear graphs from given equations • Finding equations from linear graphs	• Draw and find equations of non-linear functions such as parabola, hyperbola and exponential graph • Work with transformations such as vertical and horizontal shifts in the above mentioned graphs • Domain and range of functions • Inverses of functions including logarithmic graphs

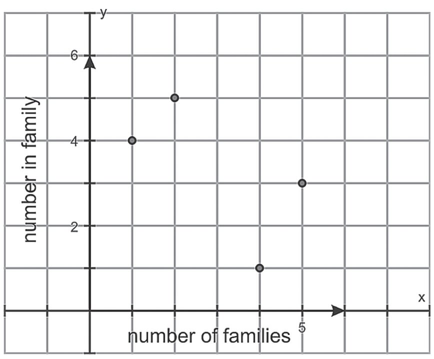
GLOSSARY OF TERMS

Term	Explanation / Description
Linear graph	This is a graph of a linear function, where all plotted points lie on a straight line.
Discrete data	Separate or distinct items or groups of data. For example: The number of people in our class… this data can be counted.
Continuous data	This is data with no gaps. For example: measuring a tree will be continuous data.
Independent variable	The variable that determines what happens to the other variable in an equation. In graphing this is the x variable.
Dependant variable	This is the changing variable that is altered by the function that contains the independent variable this would be the y variable.
Linear	Straight line
Non- linear	Curved line
Cartesian plane	A plane formed by the intersection of the x and y axis. We draw graphs on these planes and connect coordinates to form linear and non-linear functions.
Axes	These are straight lines joined at the origin, these are used to determine the exact points required to draw functions.
Origin	The intersection of the axes (0;0) is the coordinate that represents the origin.
Coordinate	This is a unique ordered pair of numbers that identifies a point on the Cartesian plane. The first number in the ordered pair identifies the position with regard to the x axis and then the second identifies the relation to the y axis.
Abscissa	The first element of the co-ordinate (x value)
Ordinate	The second element of the co-ordinate (y value)
Gradient	The slope of the line at a particular point. This is the change in y over change in x ($\frac{\text{vertical change}}{\text{horizontal change}}$)
Intercept	The line cutting the axes
Parallel	Lines that have a constant distance between them. Parallel lines can never meet/cross.
Perpendicular	To meet at a right angle. Lines are perpendicular when they cross at a right angle.
Maximum	The highest point the graph goes to (linked to the y-value)
Minimum	The lowest point the graph goes to (linked to the y-value)

SUMMARY OF KEY CONCEPTS

Analyse and interpret graphs

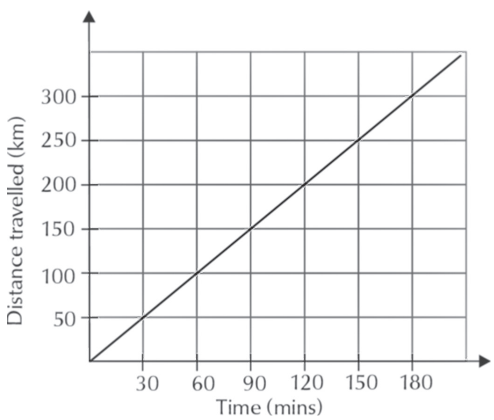
1. Graphs are used often to represent real situations and it is important that learners can read and understand them.
2. Graphs can be discrete or continuous. A discrete graphs consists of points (not joined) and a continuous graph consists of a solid line.
3. Whether discrete or continuous, two axes are always used. The horizontal axis represents the independent variable and the vertical axis represents the dependant variable. Time is often involved in these types of graphs and time is always independent. (No-one can change time).
4. For example:



   a. A discrete graph:

Note that it would not make sense to join these points as there can't be 2,5 families or 3,2 children etc.

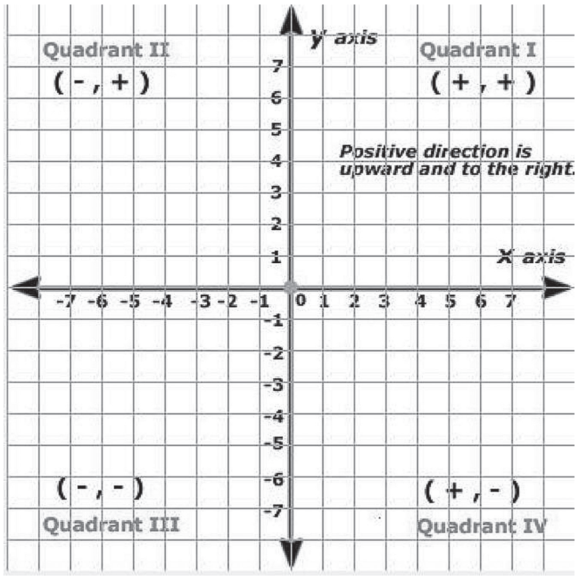
   b. A continuous graph:



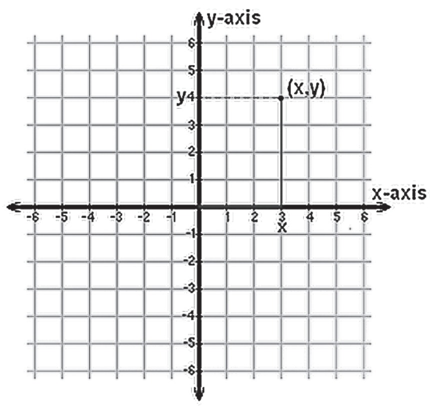
Note now that a point between the main points could be read off the graph and still make sense. For example, at 70 minutes, the distance travelled was approximately 115km.

The Cartesian Plane

1. The Cartesian Plane is comprised of various parts that enable us to visually represent functions whether they are continuous or discrete. A set of ordered pairs (co-ordinates) is made of a (x;y) value. The x value rests along a horizontal line and the y value is found along a vertical line.



2. Plotting points on the Cartesian plane requires that we plot a single point by combining the x value of the co-ordinate and the y value of the co-ordinate.



Teaching Tip:
For learners who may get confused with which number comes first, remind them that it is in alphabetical order. The x-value is always first and the y-value is always second.

NOTE: It is important for the learners to spend time practicing plotting points correctly. If this isn't achieved at a high level, the drawing of graphs this year and in future years will be increasingly difficult for them. A good way to practice is by plotting points that form a picture. They are easy for you as the teacher to check and the learners can also see if something doesn't look correct. There are two such tasks in the resource section at the end of the section

Topic 4 Graphs

Linear graphs

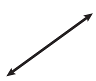
1. Intercepts

   An intercept is a point where a graph crosses one of the axes. The x-intercept is in the form (x ;0) and the y-intercept is in the form (0 ;y)
   To find the x-intercept of any function (graph), we need to make y = 0
   Why? Because no matter which point we consider on the x-axis, the y-coordinate will always be zero
   To find the y-intercept of any function we need to make x = 0
   Why? Because no matter which point we consider on the y-axis, the x-coordinate will always be zero

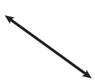
   For Example: For the function, $y = 2x + 4$ the intercepts would be found as follows:
   $x -$ intercept: $y = 0$
   $0 = 2x + 4$ Now solve for x
   $-4 = 2x$
   $-2 = x$ The $x-$intercept is $(-2\ ;\ 0)$
   $y -$ intercept: $x = 0$
   $y = 2\ (0) + 4$
   $y = 0 + 4$
   $y = 4$ The $y -$ intercept is $(\ 0\ ;\ 4)$
   These two points can be plotted and joined to form the straight line represented.

2. Gradient

   The standard form of the straight line function is always $y = mx + c$ or $y = ax + q$
   The ‘m’ or ‘a’ always represents the gradient of the line. This value tells you which way the line will lie (up or down) and how steep it will be.
   If the m-value is positive the line lies in this direction

   
   (sloping upwards left to right)

   If the m-value is negative the line lies in this direction

   
   (sloping downwards from left to right)

   If the gradient is $\frac{2}{3}$ this means that from any point on the graph, you would need to go two units up and 3 units across (to the right) to meet up with the graph again. It is commonly known as rise over run ($\frac{rise}{run}$).
   To find the gradient of a line when the function is not given, two points are required. $(x_1;y_1)$ and $(x_2;y_2)$. Using these co-ordinates, the following formula is used:

   $$m = \frac{y_2 - y_1}{x_2 - x_1}$$

For example: Find the gradient of a line that goes through the following points:
(-1 ; 4) and (3 ; 2)

Teaching Tip:
Encourage learners to label the points. Call the first one $(x_1;y_1)$ and the second $(x_2;y_2)$.
Remind learners that they will be using their substitution skills from the algebra section.

Solution:
$$m=\frac{2-4}{3-(-1)}$$
$$m=\frac{-2}{3+1}$$
$$m=\frac{-2}{4}$$
$$m=\frac{-1}{2}$$

This tells us that the line slopes downwards and that for every 1 unit we go down, we would need to go across (right) two units to get back to the line.

Teaching Tip:
Encourage learners to draw a sketch to plot these two points and check if their answer looks right. In other words – does it slope downwards as the gradient implies and could they start at any point on the line and move down one and across two and be back on the line again.If the gradients of two lines are the same, the lines will be parallel.

Drawing linear graphs from given equations

1. In order to draw a linear graph there are a few methods. The most common one being the dual-intercept method.
2. This method involves finding the x-intercept and the y-intercept (dual = 2). Once these have been found, the two points can be plotted, then joined with a ruler to form the straight line.

   It is important to remember to extend the line past the two points as it is a continuous graph and many points could lie between the intercepts and past them.
3. For example: Sketch the graph $y = 2x + 1$

   $x -$ intercept: $y = 0$

   $0 = 2x + 1$ Now solve for x

   $-1 = 2x$

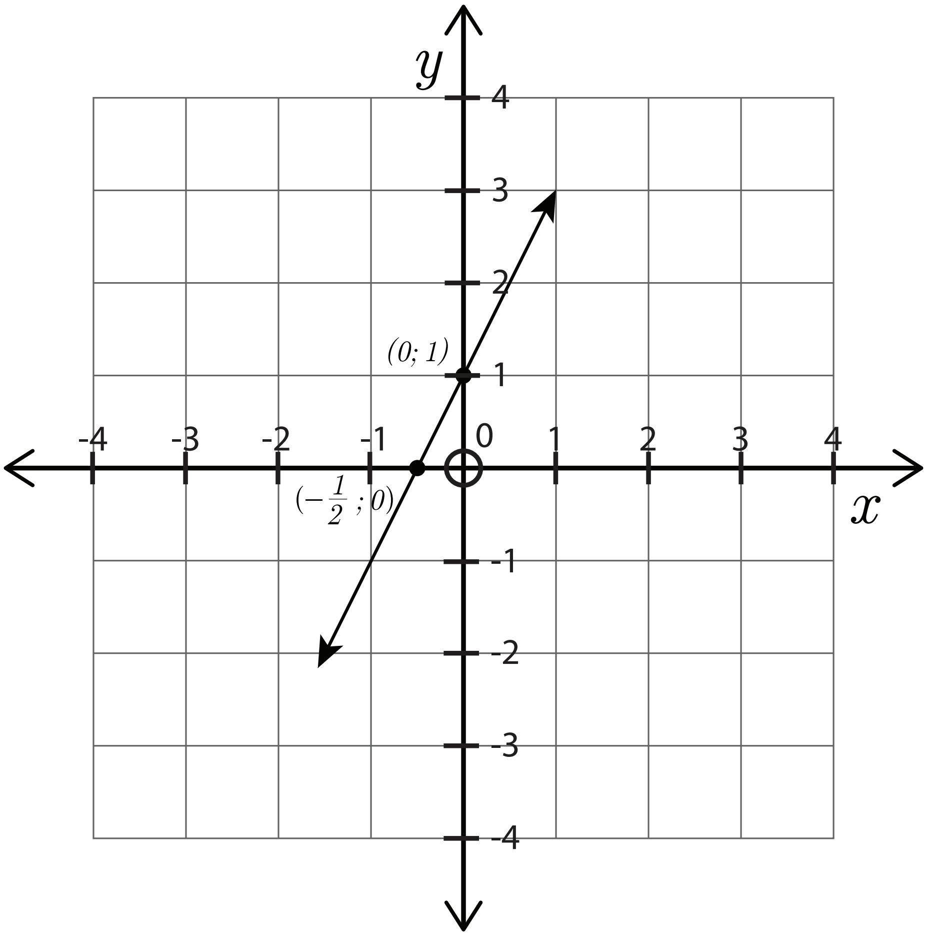
   $-\frac{1}{2} = x$ The $x -$ intercept is $(-\frac{1}{2}\ ;0)$

   $y -$ intercept: $x = 0$

   $y = 2(0) + 1$

   $y = 0 + 1$

   $y = 1$ The $y -$ intercept is $(0\ ;1)$



4. Another method used is the table method. This involves drawing up a table, putting in some $\mathrm{x}-$ values then using substitution to find the corresponding $\mathrm{y}-$ values.
For example: Draw the graph $\mathrm{y}=2\mathrm{x}+3$ using the table method
First draw up a table and put in a few $\mathrm{x}-$ values

x	-3	-2	-1	0	1
y					

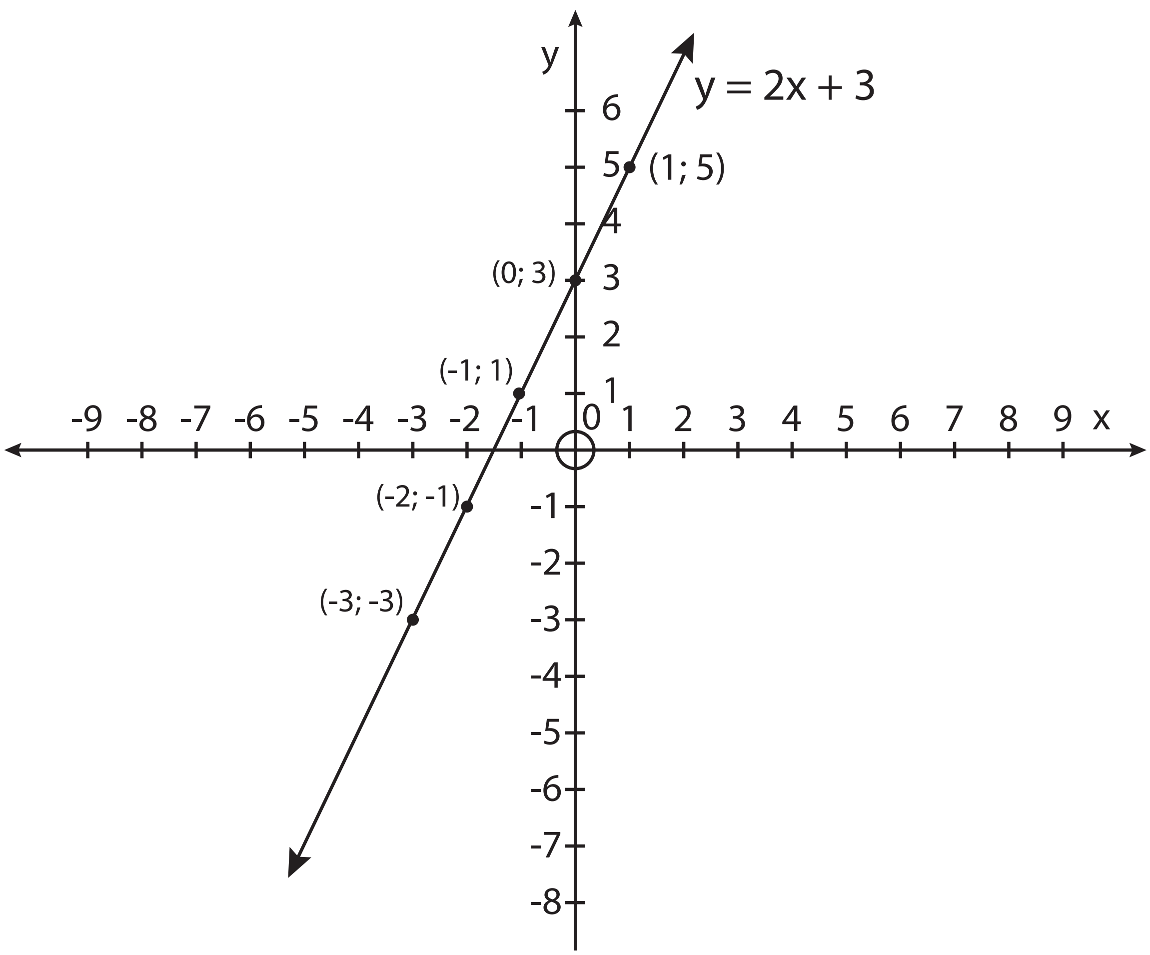
Use substitution to find the y-values and fill them in on the table

$y=2x+3$ $y=2(-3)+3$ $y=-3$	$y=2x+3$ $y=2(-2)+3$ $y=-1$	$y=2x+3$ $y=2(-1)+3$ $y=1$	$y=2x+3$ $y=2(0)+3$ $y=3$	$y=2x+3$ $y=2(1)+3$ $y=5$

The following co-ordinates now make up the points that will form the straight line:

(-3 ; -3) (-2 ; -1) (-1 ; 1) (0 ; 3) (1 ; 5)

Draw a Cartesian plane and plot the points:



Finding the equation of a linear graph

1. When given points that lie on a graph we need to be able to find the equation of that line
2. There are two types:
   a. Given the y-intercept and one other point
   Example: determine the equation for the line given the following points on that line. (0;2) and (3;6)
   The point (0;2) is the y-intercept and thus it is the c-value.

   $$y = mx + 2$$

   Then substitute the other point into the equation to solve for m. (The gradient)

   $$\begin{aligned} 6 &= m(3) + 2 \\ 6 - 2 &= 3m \\ 4 &= 3m \\ \therefore m &= \frac{4}{3} \\ \therefore y &= \frac{4}{3}x + 2 \end{aligned}$$

   b. Given two points
   Example: determine the equation of the line given that (1;8) and (-4;16) are points on the line.
   First use the two points to find the gradient.

   $$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ m &= \frac{16 - 8}{-4 - 1} \\ m &= -\frac{8}{5} \end{aligned}$$

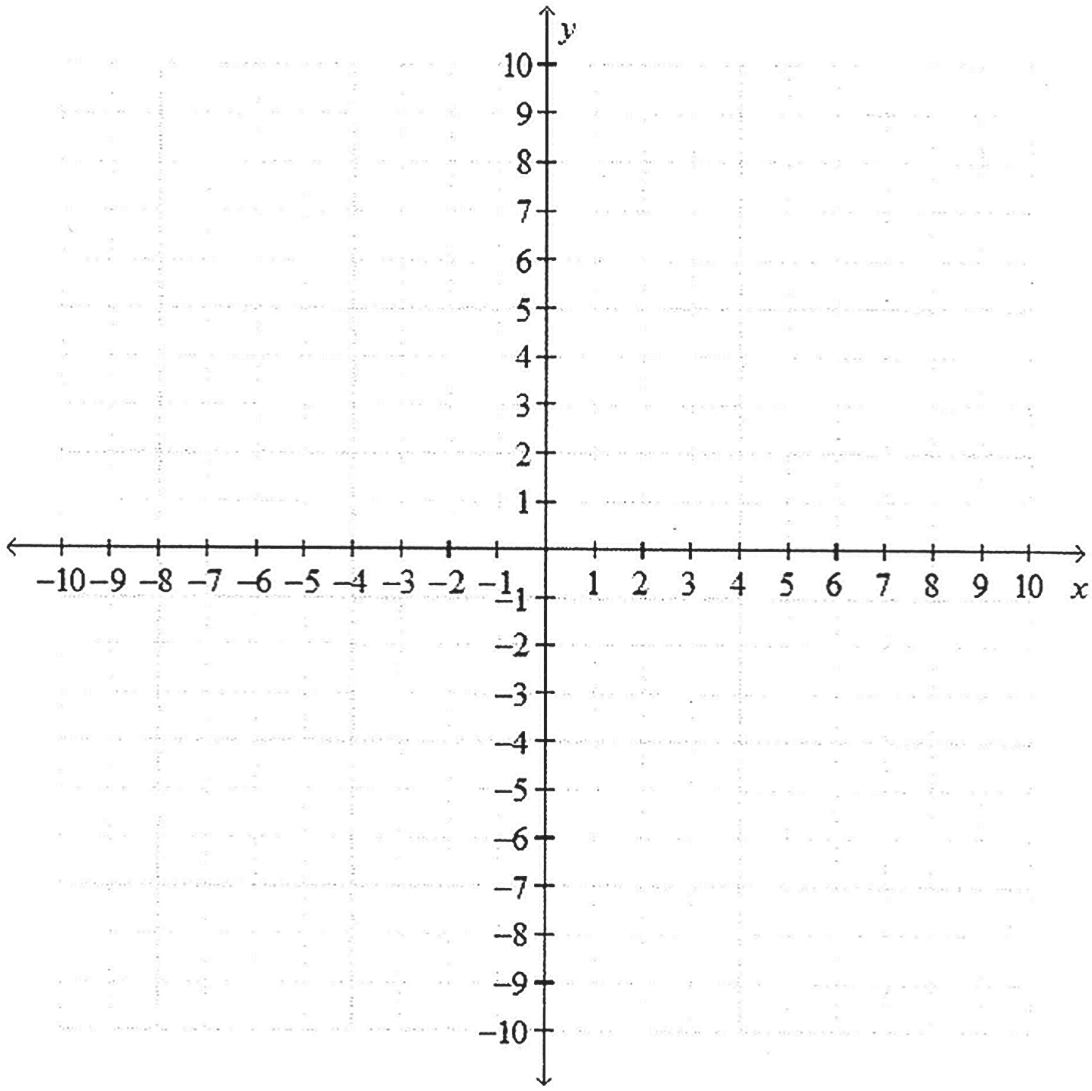
   Put the value found into the equation

   $$y = -\frac{8}{5}x + c$$

Substitute any one of the points into the function and solve for c.
Using (1 ; 8):

$$\begin{aligned} 8 &= -\frac{8}{5}(1) + c \\ 40 &= -8 + 5c \\ 48 &= 5c \\ c &= \frac{48}{5} \\ \therefore c &= 9\frac{3}{5} \\ \therefore y &= -\frac{8}{5}x + 9\frac{3}{5} \end{aligned}$$

RESOURCES

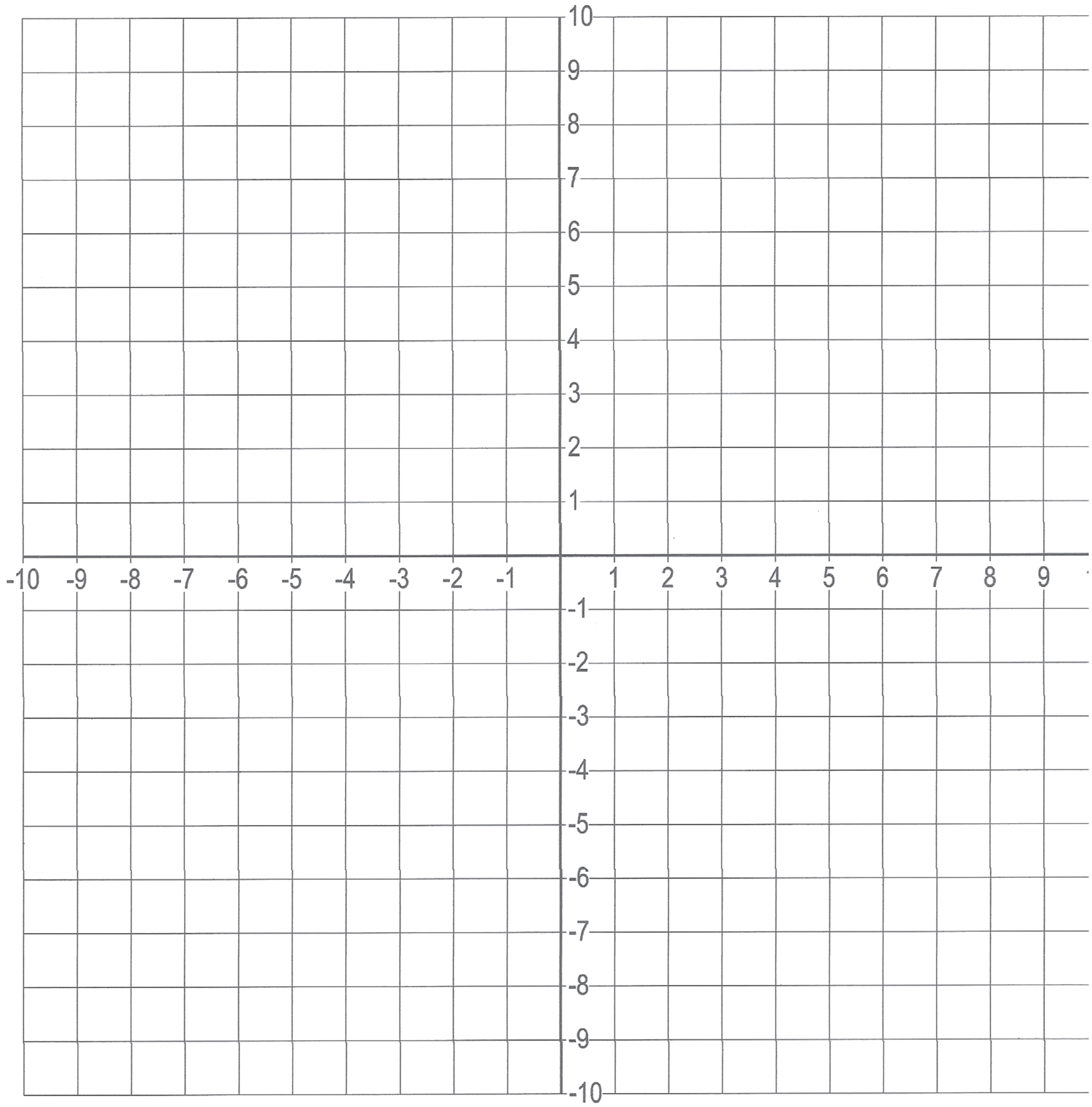


Plot the following points. Join the points in alphabetical order.

A(-6 ; -3)	B(-3 ; -1)	C(2 ; 1)
D(5 ; -1)	E(5 ; -4)	F(6 ; 0)
G(3 ; 3)	H(4 ; 4)	I(5 ; 4)
J(3 ; 5)	K(2 ; 4)	L(0 ; 6)
M(-5 ; 5)	N(-1; 5)	O(0 ; 3)
P(-3 ; 0)	Q(-7 ; -1)	R(-4 ; -1)
A(-6 ; -3)		

(This is a bird in flight)

There is a picture hidden in this grid. Connect the points with lines to reveal it.



Line 1: (-2,3), (-3,2), (-3,0) **Line 2:** (1,2), (3,2), (4,1), (9,6), (7,8), (-7,-6), (-9,-4), (-4,1), (-3,0), (-1,0) **Line 3:** (3,2), (3,-2) **Line 4:** (-1,2), (-1,4), (0,5), (-5,10), (-7,8), (7,-6), (5,-8), (0,-3), (1,-2), (1,0) **Line 5:** (-3,-2), (-3,-10), (3,-10), (3,-6) **Line 6:** (1,-6), (1,-8), (2,-8), (2,-6), (1,-6) **Line 7:** (2,3), (1,4), (-1,4) **Line 8:** (-2,-3), (-1,-3), (-1,-5), (-2,-5), (-2,-3)

(This is a windmill)

TOPIC 5: SURFACE AREA AND VOLUME OF 3D OBJECTS

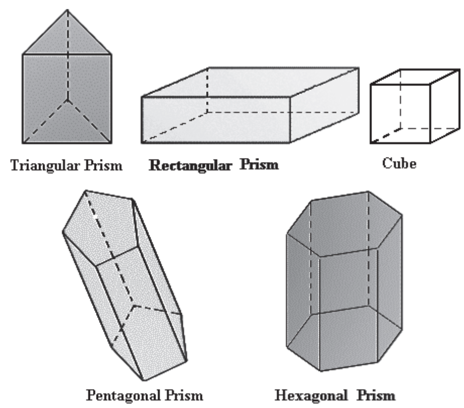
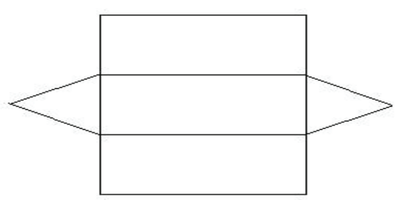
INTRODUCTION

- This unit runs for 5 hours.
- It is part of the Content Area, 'Measurement', and counts for 10 % in the final exam.
- The unit covers the use of appropriate formulae to find surface area, volume and capacity of prisms and cylinders.
- The purpose of teaching surface area is, in general, to find how much material would be needed to make a 3D shape (useful in deciding what the best container would be for packaging a type of food). This would include knowing how much space is used on the surfaces of all the shapes making up a 3D object (useful if those areas need painting for instance).
- The purpose of teaching volume would be either to find how one or many objects could fit into a certain space or how much liquid a 3D object could hold (capacity).

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 8	GRADE 9	GRADE 10/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Calculate surface area, volume and capacity of cubes, rectangular prisms, triangular prisms • Describe the interrelationship between surface area and volume • Solve problems involving surface area, volume and capacity • Use and convert between appropriate SI units, including: $mm^2 \leftrightarrow cm^2 \leftrightarrow m^2 \leftrightarrow km^2$ $mm^3 \leftrightarrow cm^3 \leftrightarrow m^3$ $ml\,(cm^3) \leftrightarrow l \leftrightarrow kl$	• Solve problems using appropriate formulae and conversions between SI units • Calculate the surface area, volume and capacity of cubes, rectangular prisms, triangular prisms and cylinders • Investigate how doubling any or all the dimensions of right prisms and cylinders affects their volume	• Solve problems involving volume and surface area of solids studied in earlier grades as well as spheres, pyramids and cones and combinations of those objects.

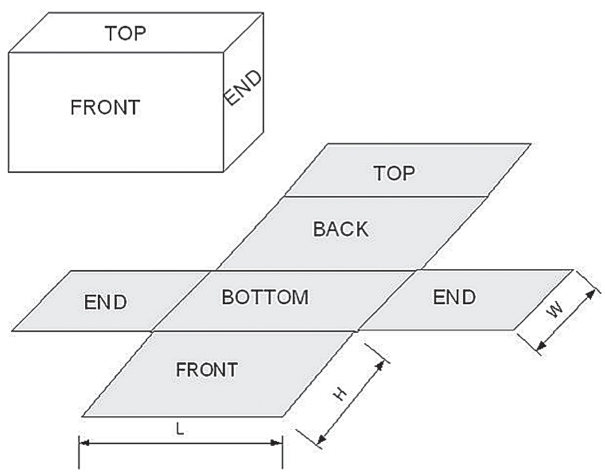
GLOSSARY OF TERMS

Term	Explanation / Diagram
2D	2-dimensional
3D	3-dimensional
Polygon	A 2D shape in which all the sides are made up of line segments. A polygon is given a name depending on the number of sides it has. Example: A 5 sided polygon is called a pentagon
Solid	An object that occupies space (3-dimensional)
Polyhedron	A solid in which all the surfaces (faces) are flat.
Prism	A solid with parallel equal bases. The bases are both polygons.
Right Prism	A prism which has the sides at right angles to the base. 
Face	A flat surface of a prism
Edge	Where the faces of a prism meet
Vertex	Where the edges of a prism meet (the corner)
Cube	A solid with six equal square faces
Cuboid/Rectangular Prism	A solid with six rectangular faces
Triangular prism	A solid with two equal triangular faces (one is the base) and three rectangular faces
Cylinder	A solid with two equal circular faces (one is the base) and one rectangle (curved)
Net	A 2D shape, that when folded forms a 3D shape. For example:  This is a net of a triangular prism.
Surface Area	The area taken up by the net of a 3D solid. The sum of the area of all the faces.
Volume	The space taken up by a 3D solid. To find volume, the area of the base is multiplied by the perpendicular height. This only works for right prisms.
Capacity	The amount of liquid a 3D shape can hold. It is directly linked to volume.

SUMMARY OF KEY CONCEPTS

Surface Area

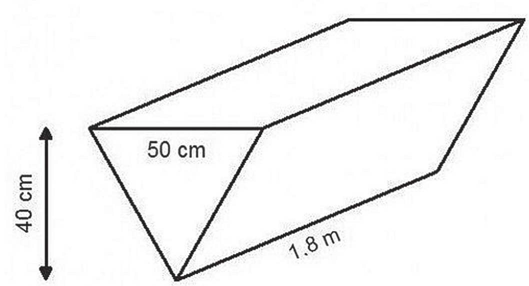
1. The term 'surface area' is linked to 3-dimensional objects only. (When dealing with 2-dimensional shapes the term 'area' is used)
2. The answer will always be in square units. For example, cm^2
3. To find the surface area of a 3D object is to find the total area taken up by the net of the 3D shape (what the 3D shape looks like in its flattened form)
   For Example:
   When finding the surface area of a rectangular prism, the area of each of the 6 rectangles need to be found and added together.



   (The Grade 8 workbook Term 3 has examples you can also look at)
4. When asked a surface area question it is important to be clear on whether the shape is a closed figure or not. In other words, it may be a box without a lid. This will affect how answering the question should be approached.

   For example: Consider this diagram of a water trough made in the shape of a triangular prism:

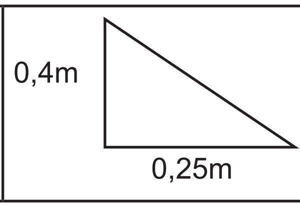
   To find how much metal should be used to make the trough would be the same as being asked to find the surface area. However, it needs to be noted that it is open at the top and therefore missing the rectangle that would have been at the top to form a closed triangular prism.



Teaching Tip:
Encourage learners to draw a net of the shape and to write down what shapes make up the 3D shape being used. Measurements should also be filled in on the drawing of the net. Note that the measurements are given in both centimetres and metres and that this has been put onto the diagram in both. Once learners start answering the question and doing calculations, they will need to decide which of the two measurements will be used.

Solution:

Surface Area = Area of two triangles + area of two rectangles

$h^2 = (0,25)^2 + (0,4)^2$ $h^2 = 0,0625 + 0,16$ $h^2 = 0,2225$ $h = 0,472$	

Note: The side of the triangle that is the same as the breadth of the rectangle is missing. The theorem of Pythagoras needs to be used to calculate this. The perpendicular height forms a right-angled triangle.

Teaching Tip:
Learners should fill this new piece of information on the diagram
Point out to learners how common it is that something they have learnt in the past or in another topic is often used in another area. It is always essential to understand each topic in isolation but at the same time be ready to use it elsewhere as well.

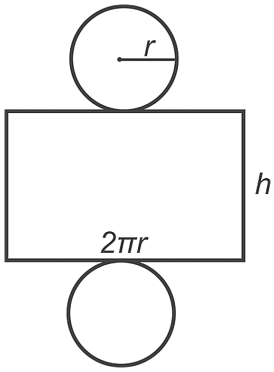
$$= 2(\frac{1}{2}\text{b}\times\text{h}) + 2(\text{l}\times\text{b})$$
$$= 2(\frac{1}{2}(0,5\text{m})(0,4\text{m}) + 2(1,8\text{m})(0,472\text{m})$$
$$= 0,2\text{m}^2 + 1,699\text{m}^2$$
$$= 1,899\text{m}^2$$

To find the surface area of a cylinder, consider what it looks like in its 2D form (its net)

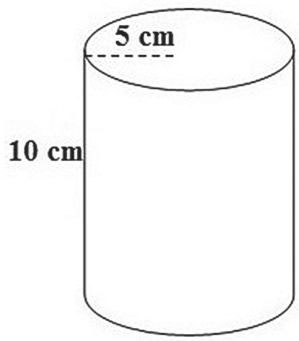
Notice that the length of the rectangle which forms the curved part of the cylinder is $2\pi r$.

Explain to learners that this is due to the fact that the length of the rectangle matches up with the circumference of the circle when the cylinder is put back together.

Therefore, to find the surface area of a cylinder we need to find the area of two circles and a rectangle and add the answers together.



For example: Find the surface area of the following cylinder;



$$\begin{aligned}\text{Surface Area} &= 2\pi r^2 + (2\pi r \times h) \qquad (2 \text{ circles plus one rectangle})\\ &= 2\pi(5\text{cm})^2 + (2\pi(5\text{cm})(10\text{cm}))\\ &= 157{,}08\text{cm}^2 + 314{,}16\text{cm}^2\\ &= 471{,}24\text{cm}^2\end{aligned}$$

Volume

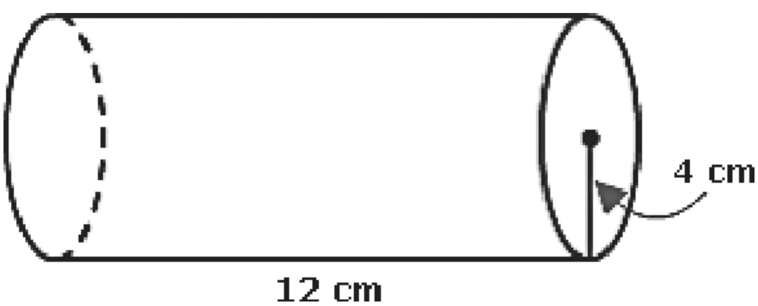
1. The term 'volume' is linked to 3-dimensional objects only.
2. The answer will always be in cubic units. For example, cm^3
3. To find the volume of any right prism, the basic formula is:

   Area of base x perpendicular height
   Notice again that you are required to know how to find the area of the basic shapes (square, rectangle and triangle).
   Note: There are examples of the three basic prisms (cube, cuboid and triangular prism) in the Term 3 Grade 8 workbook

VOLUME OF:	AREA OF BASE x HEIGHT
Cube	$(l \times l) \times ht$ $= l \times l \times l$ $= l^3$
Rectangular prism (cuboid)	$(l \times b) \times h = lbh$
Triangular prism	$(\frac{1}{2} b \times h) \times H$ Note: The first 'h' represents the height of the triangle which is required in order to find the area of the base. The 2nd 'H' represents the height of the prism.
Cylinder	$\pi r^2 \times ht$ $= \pi r^2 ht$

4. Finding the volume of a cylinder is easier than finding the surface area. The same general formula is used as for the other right prisms. Namely, Area of base x perpendicular height.

For example:
Find the volume of the following cylinder:



$$\text{Volume} = \pi r^2 ht$$
$$= \pi (4\text{cm})^2 (12\text{cm})$$
$$= 603,19\text{cm}^3$$

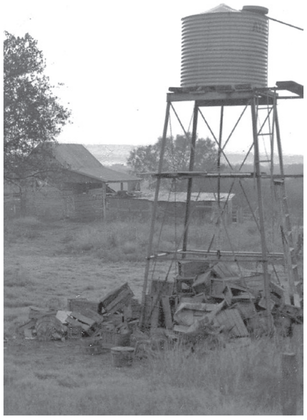
Capacity

1. Capacity is how much liquid a 3D shape (solid) can hold. It is directly linked to volume.
2. The following three conversions should be learnt:

$$1cm^3 = 1ml$$
$$1000cm^3 = 1000ml = 1l$$
$$1m^3 = 1000l = 1kl$$

3. For example:
   A teaspoon holds 5millilitres. This means its size is $5cm^3$
   A carton of fruit juice holds 1 litre (1 000 millilitres). This means its size is $1000cm^3$.
4. Remember that capacity is not the same as volume. It is directly linked to it and converts amount of space a shape takes up to how much liquid it can hold.
5. For example:

On a farm there is a cylindrical tank for water.



It has a diameter of 1m and is 1,5m in height. Find how much water it can hold.
Firstly, find the volume of the tank.
Note that the diameter was given and as the radius is needed this needs to be divided by 2.

$$\begin{aligned} Volume &= \pi r^2 ht \\ &= \pi(0,5m)^2(1,5m) \\ &= 1,18m^3 \end{aligned}$$

Secondly, convert this answer to capacity. Remember that

$$1\text{m}^3 = 1000\text{l} = 1\text{kl}$$

$$1,18\text{m}^3 = 1180\text{litres} \text{ (or } 1,18 \text{ kl)}$$

Conversions

Although part of this has already been covered in the notes on Area and perimeter of 2D shapes in Term 2 it is worth looking at again.

1. Linear measurement

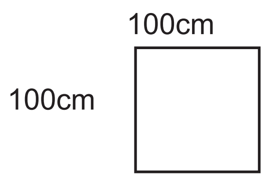
   Consider the length of a line:

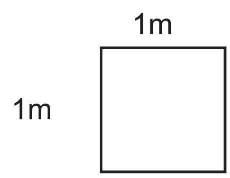
   1m = 100cm and 1cm = 10mm and 1km = 1000m

2. Area measurements

   When converting between units of area it isn't as straightforward as multiplying or dividing by 10, 100 or 1000 as it is for linear measurements.

   Consider this square:





$$\begin{aligned} Area &= 100cm \times 100cm \\ &= 10000cm^2 \end{aligned}$$

$$\begin{aligned} Area &= 1m \times 1m \\ &= 1m^2 \end{aligned}$$

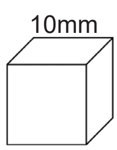
Since $1\text{m} = 100\text{cm}$, these squares are the same size, so therefore

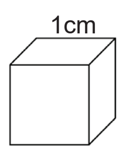
$$10000\text{cm}^2 = 1\text{m}^2$$

Normally, we would think of the conversion $1\text{m} = 100\text{cm}$, but since we are dealing with area we need to remember that

$$1\text{m}^2 = 100\text{cm} \times 100\text{cm} = 10000\text{cm}^2$$

3. Similarly for volume:





$$\begin{aligned} Volume &= l^3 \\ &= (10mm)(10mm)(10mm) \\ &= 1000mm^3 \end{aligned}$$

$$\begin{aligned} Volume &= l^3 \\ &= (1cm)(1cm)(1cm) \\ &= 1cm^3 \end{aligned}$$

As the cubes are the same size $(10\text{mm} = 1\text{cm})$, their volumes must be the same as well

$$1000\text{mm}^3 = 1\text{cm}^3$$

Normally we would think of the conversion $1\text{cm} = 10\text{mm}$, but since we are dealing with volume we need to remember that

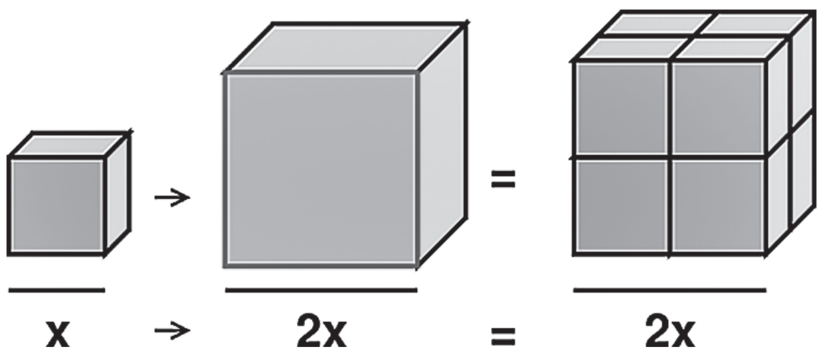
$$1\text{cm}^3 = 10\text{mm} \times 10\text{mm} \times 10\text{mm} = 1000\text{mm}^3$$

Here is a summary of the more common conversions required:

AREA	VOLUME
$1cm^2 = 100mm^2$ (10×10)	$1cm^3 = 1000mm^3$ $(10 \times 10 \times 10)$
$1m^2 = 10000cm^2$ (100×100)	$1m^3 = 1000000cm^3$ $(100 \times 100 \times 100)$
$1km^2 = 1000000m^2$ (1000×1000)	$1km^3 = 1000000000m^3$ $(1000 \times 1000 \times 1000)$

The effect of doubling dimensions on volume and surface area

1. When the size of an object increases, its surface area and volume will also increase.
2. If all measurements are doubled, its surface area will be 4 times larger ($2^2 = 4$)
3. If all measurements are doubled, its volume will be 8 times larger ($2^3 = 8$)
4. Consider the following cubes:



The length of the side of the original cube is x

$$\text{Surface Area} = 6 \times x \times x = 6x^2 \qquad \text{Volume} = x \times x \times x = x^3$$

The length of the side of the larger cube is $2x$

$$\text{Surface Area} = 6 \times 2x \times 2x = 24x^2 \qquad \text{Volume} = 2x \times 2x \times 2x = 8x^3$$

Note that the surface area is 4 times larger and that the volume is 8 times larger